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A Thermodynamically-based Network Model for Intermittency during Multiphase Flow in Porous Media

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Darcy's Law

Multiphase Darcy's law:

$$\vec{v_i} = \frac{K k_i^r}{\mu_i}(\Delta p_i - \rho_i \vec{g})$$

- Fluid flow through porous media is modelled using the two-phase extension of the Darcy's law.
- There is a linear relationship between pressure gradient and flow velocity.
- Interface between fluid phases at a fixed saturation is invariant.
- The flow of a phase through the porous media occurs only through an established flow pathway.

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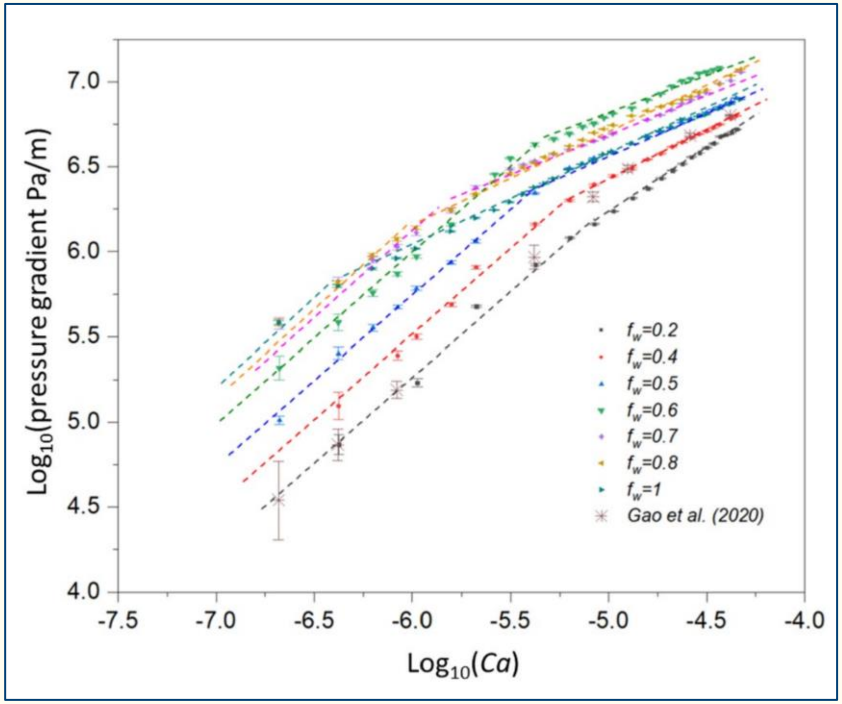
However, recent experimental studies in relation to nonlinear intermittent flow behaviours during multiphase fluid flow in porous media established the following:

- At fixed average fluid saturations, the arrangement of fluid phases is dynamic (Tallakstad et al., 2009a & 2009b).



Zhang *et. al*., 2021

- There is a transition from a linear flow regime to a nonlinear flow regime as flow rate increases (Spurin *et al*., 2019; Gao et al., 2020).

- Relationship between capillary number and pressure gradient at certain range of flow rates becomes nonlinear (Zhang *et al.,* 2021).

Fluid Intermittency

- Non-linear relationship between pressure gradient and capillary number is attributed to fluid intermittency.
- This is the periodic disconnecting and reconnecting of the non-wetting phase along flow pathways.
- Disconnection of the non-wetting phase occurs after a series of snap-off events along the flow pathways.
- Fluid intermittency is caused by the nonwetting phase periodically finding more conductive pathways through the pore space.
- The interplay of viscous and capillary forces largely determines the occurrence of intermittent flow.
- Intermittent pathway flow is controlled by:
  - Capillary number - Viscosity ratio
  - Pore geometry - Wettability

$$Ca_i = \frac{\mu_i v_i}{\sigma}$$

$$Ca = \frac{q_t}{\sigma\left(\frac{1-f_w}{\mu_{nw}} + \frac{f_w}{\mu_w}\right)}$$

$$M = \frac{\mu_{nw}}{\mu_w}$$

$$f_w = q_w / q_t$$

$$\nabla P \sim Ca^a$$

Thermodynamically-based Network Model

- An analogy between thermodynamics and immiscible fluid flow in porous media could be used to study nonlinear flow behaviours (Hansen *et al*., 2022).
- Insights will be taken from the thermodynamic formulation of multiphase flow proposed by Hansen and colleagues.
- A new traditional quasi-static pore-scale network model will be first developed.
- Modification to a probabilistic dynamic pore-scale network model will then be done.

$$P_{filling} = z \exp^{(-\frac{\Delta E}{c})}$$

$$\Delta E_{Drainage} = P_c^D - P_c^*$$

$$\Delta E_{Imb} = P_c^* - P_c^I$$

$$P_{filling} = \frac{\exp^{(-\frac{P_c^* - P_c^I}{c})}}{\exp^{(-\frac{P_c^* - P_c^I}{c})} + \exp^{(-\frac{P_c^D - P_c^*}{c})}}$$

$$z = \frac{1}{\exp^{(-\frac{P_c^* - P_c^I}{c})} + \exp^{(-\frac{P_c^D - P_c^*}{c})}}$$

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Capillary-dominated displacement (Drainage)

Circular:

$$P_c = \frac{2\sigma \cos\theta_r}{r}$$

Angular:

$$P_c = \frac{\sigma(1+2\sqrt{\pi G})\cos\theta_r F_d(\theta_r, G)}{r}$$

$$F_d(\theta_r, G) = \frac{1+\sqrt{1+4GD/cos^2\theta_r}}{1+2\sqrt{\pi G}}$$

Capillary-dominated displacement (Imbibition)

Piston-like:

$$P_c = \frac{\sigma(1+2\sqrt{\pi G})\cos\theta_A F_d(\theta_A, G)}{r} \equiv \frac{\sigma\cos\theta_A}{r} C_{It}$$

Snap-off:

$$P_c = \frac{\sigma\cos\theta_A}{r}(1-\tan\theta_A\tan\beta)$$

Pore-body filling:

$$P_c(I_n) = \frac{2\sigma\cos\theta_A}{r_p} - \sigma\sum_{i=1}^{n} b_i x_i$$

Thermodynamically-based Network Model

$$A_w = P_{filling} \times A_{w,max} + (1 - P_{filling}) \times A_{w,min}$$
$$A_{nw} = (1 - P_{filling}) \times A_{nw,max} + P_{filling} \times A_{nw,min}$$

In this proposed model, area occupied by each phase and the conductance of each phase in each element will depend on the probability of filling.

$$g_w = P_{filling} \times g_{w,max} + (1 - P_{filling}) \times g_{w,min}$$
$$g_{nw} = (1 - P_{filling}) \times g_{nw,max} + P_{filling} \times g_{nw,min}$$

In traditional pore-scale network model, all elements either have a probability of 0 (not filled) or 1 (filled).

In this proposed model, the probability is [0, 1]. The model should agree with the traditional model where there is no intermittency.

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Quasi-static Network Model

Major milestones

Pore network extraction from x-ray images → Single phase flow computations → Two phase Drainage Simulation → Two phase Imbibition Simulation

Single phase computations

Element area → Conductance per unit length → Pressure distribution → Absolute permeability

Two phase computations

Half angles → Capillary pressure → Filling → Phase area → Conductance per unit length → Pressure distribution → Relative permeability

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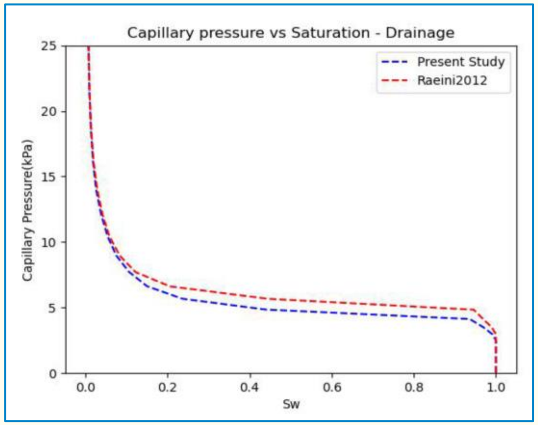
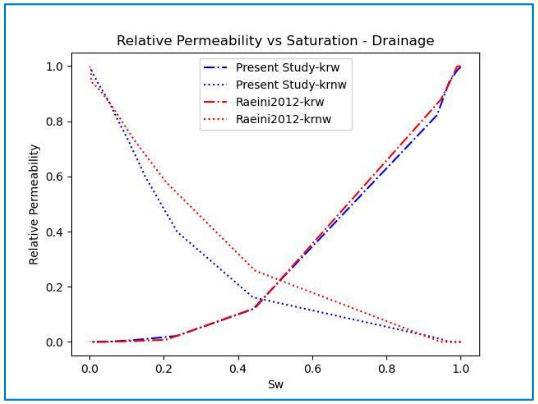
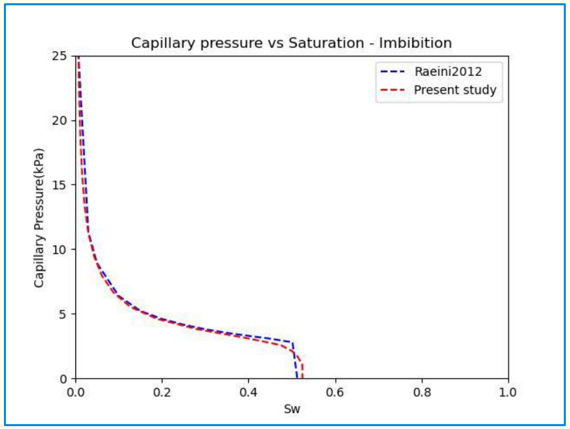
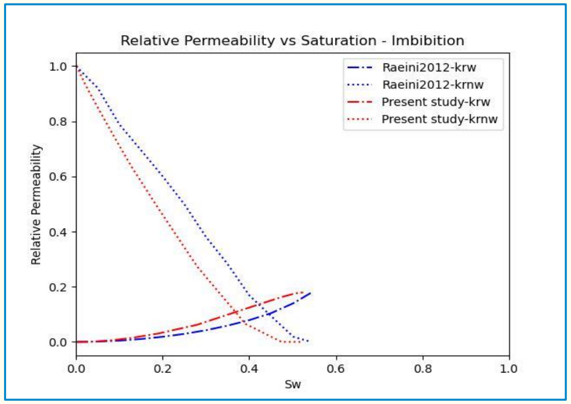
Simulation results
quasi-static network model





Primary drainage results





Secondary imbibition results

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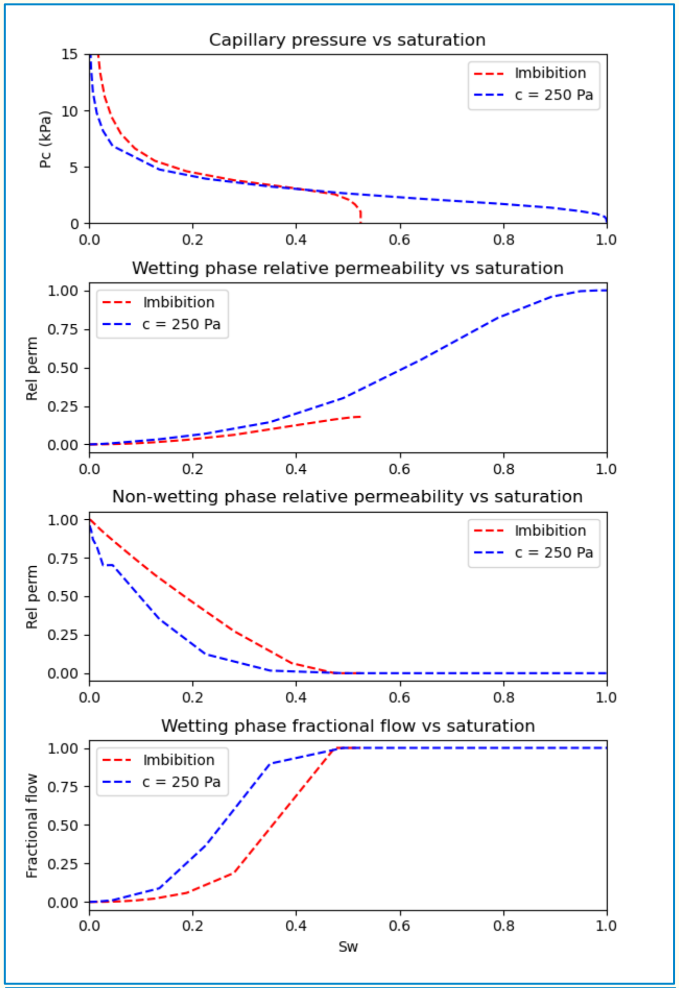
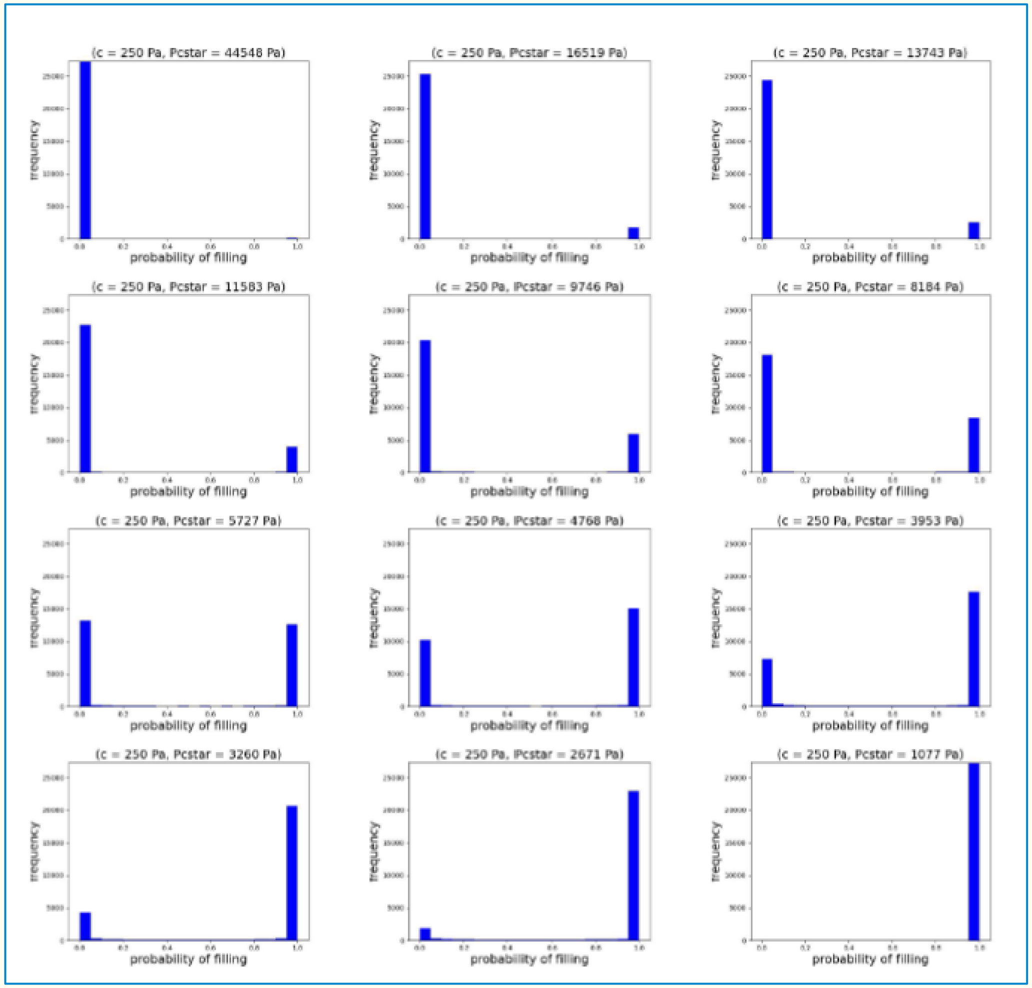
Simulation results

Probability filling when c = 250 Pa



Plots of Pc, kr and fw against Sw



Probability distribution for filling at the different values of P_c*

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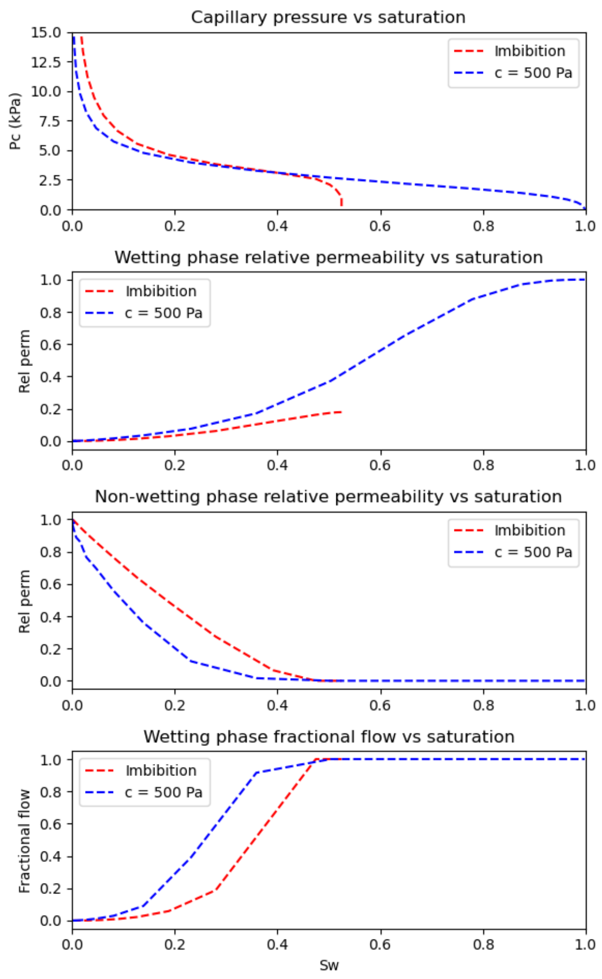
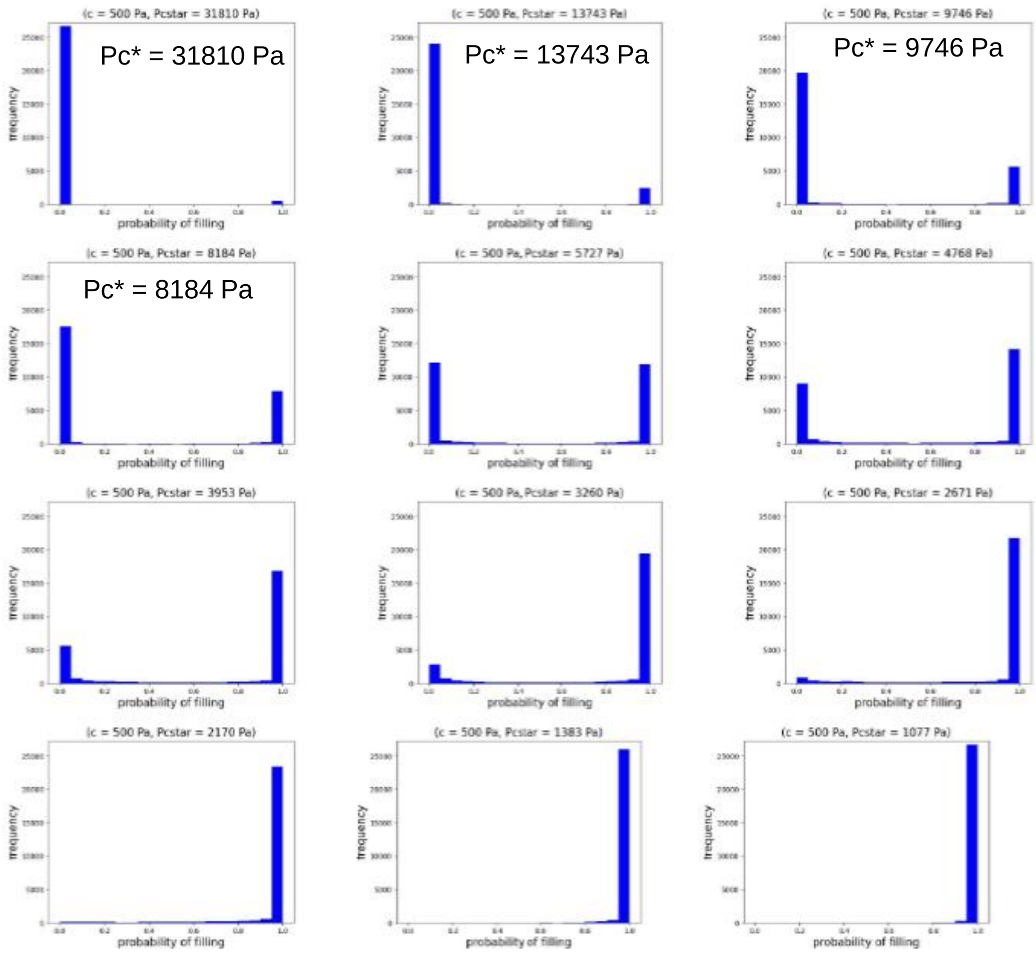
Simulation results

Probability filling when c = 500 Pa



Plots of Pc, kr and fw against Sw



Probability distribution for filling at the different values of P_c*

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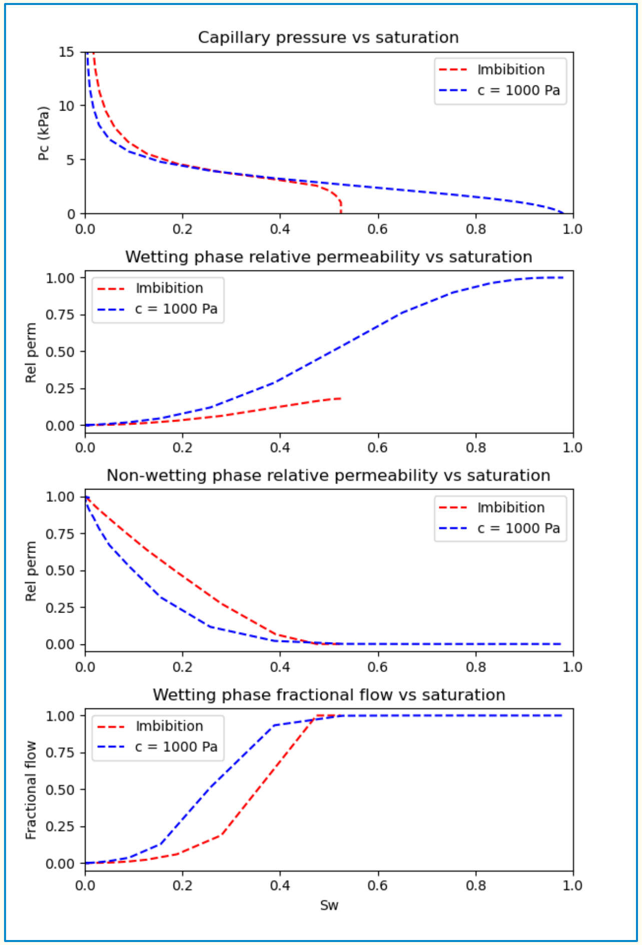
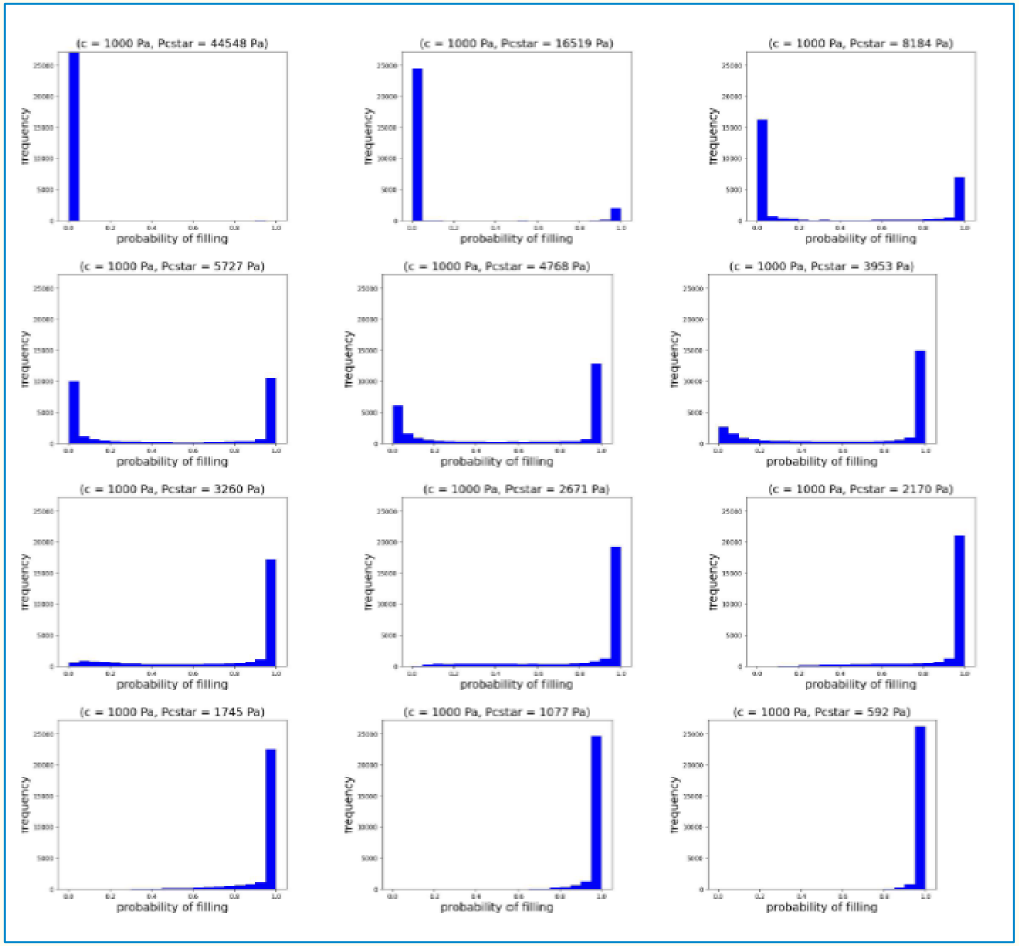
Simulation results

Probability filling when c = 1000 Pa



Plots of Pc, kr and fw against Sw



Probability distribution for filling at the different values of P_c^*

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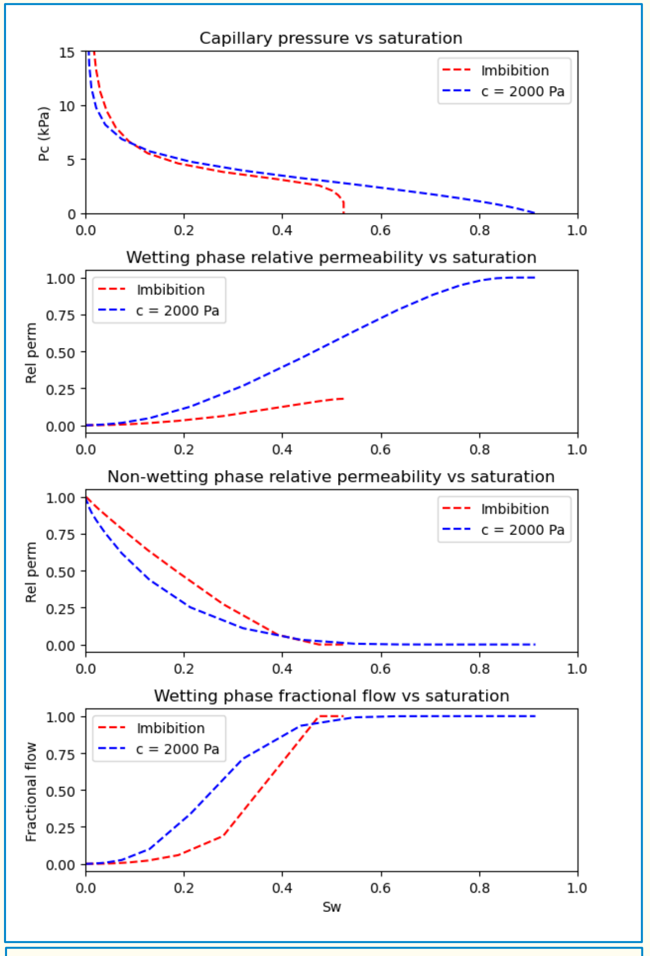
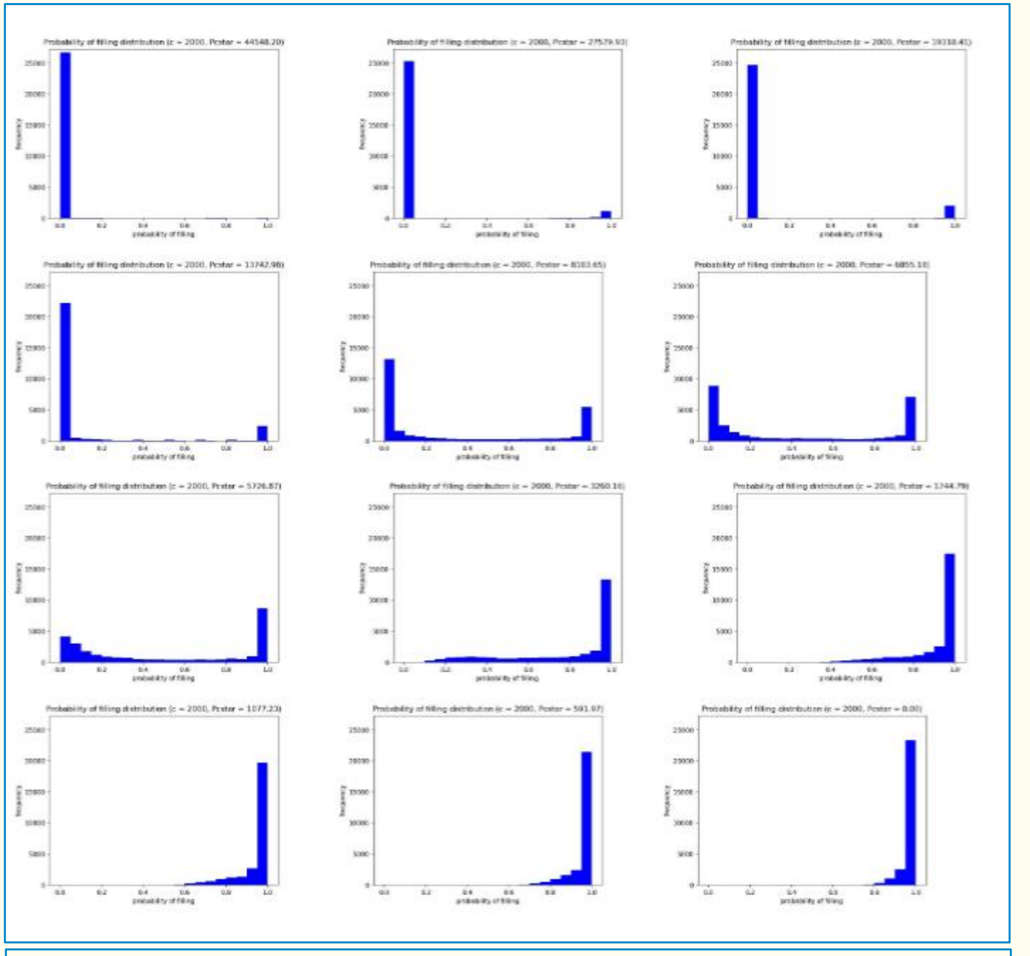
Simulation results

Probability filling when c = 2000 Pa



Plots of Pc, kr and fw against Sw



Probability distribution for filling at the different values of P_c^*

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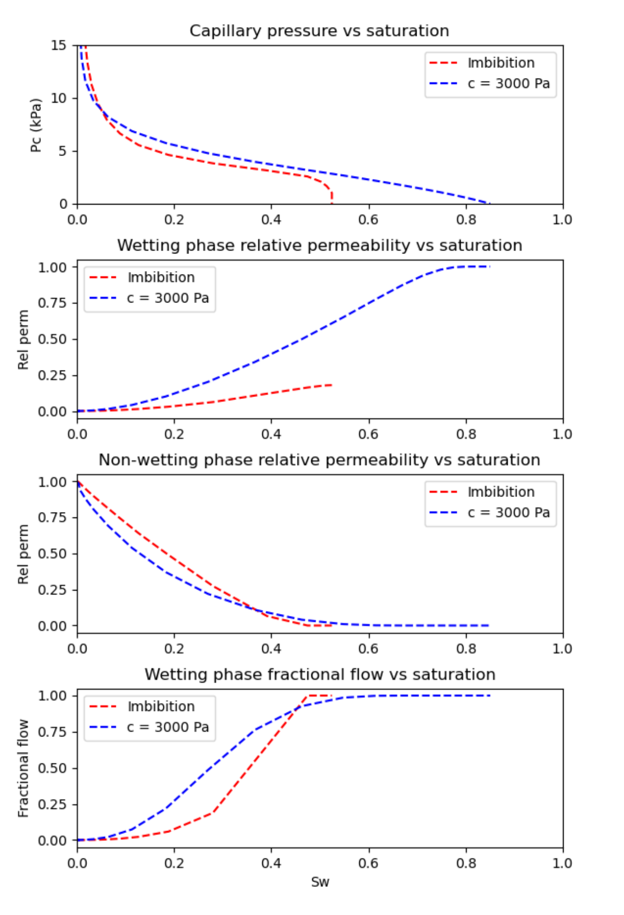
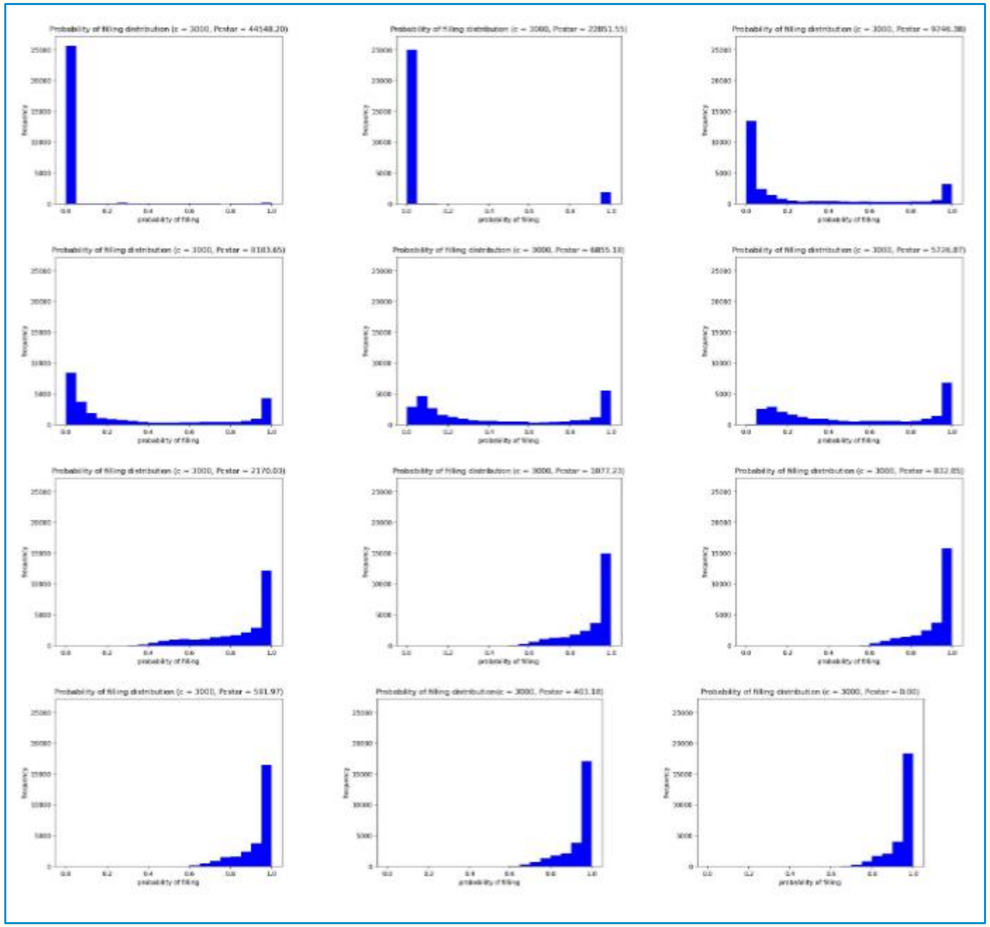
Simulation results

Probability filling when c = 3000 Pa



Plots of Pc, kr and fw against Sw



Probability distribution for filling at the different values of P_c^*

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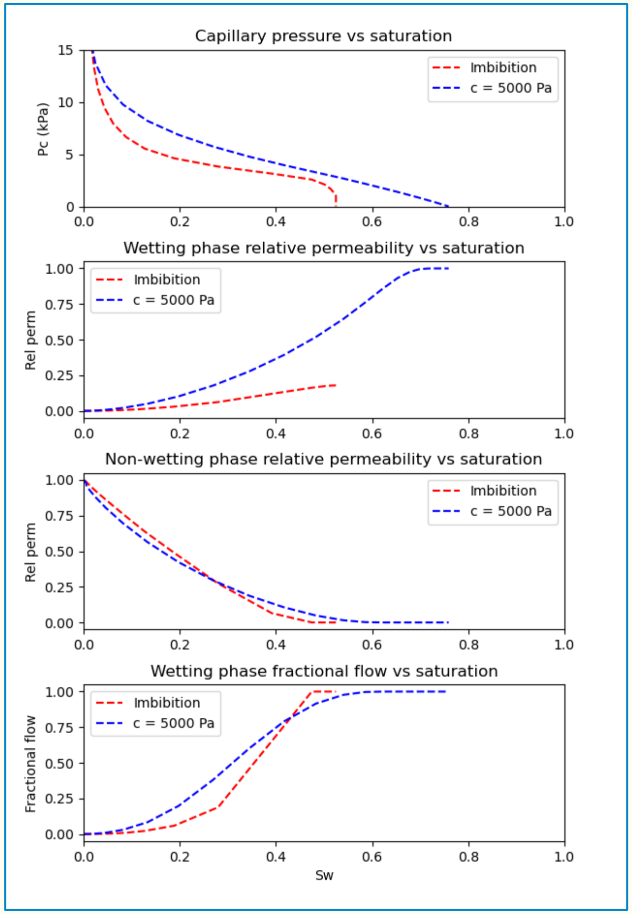
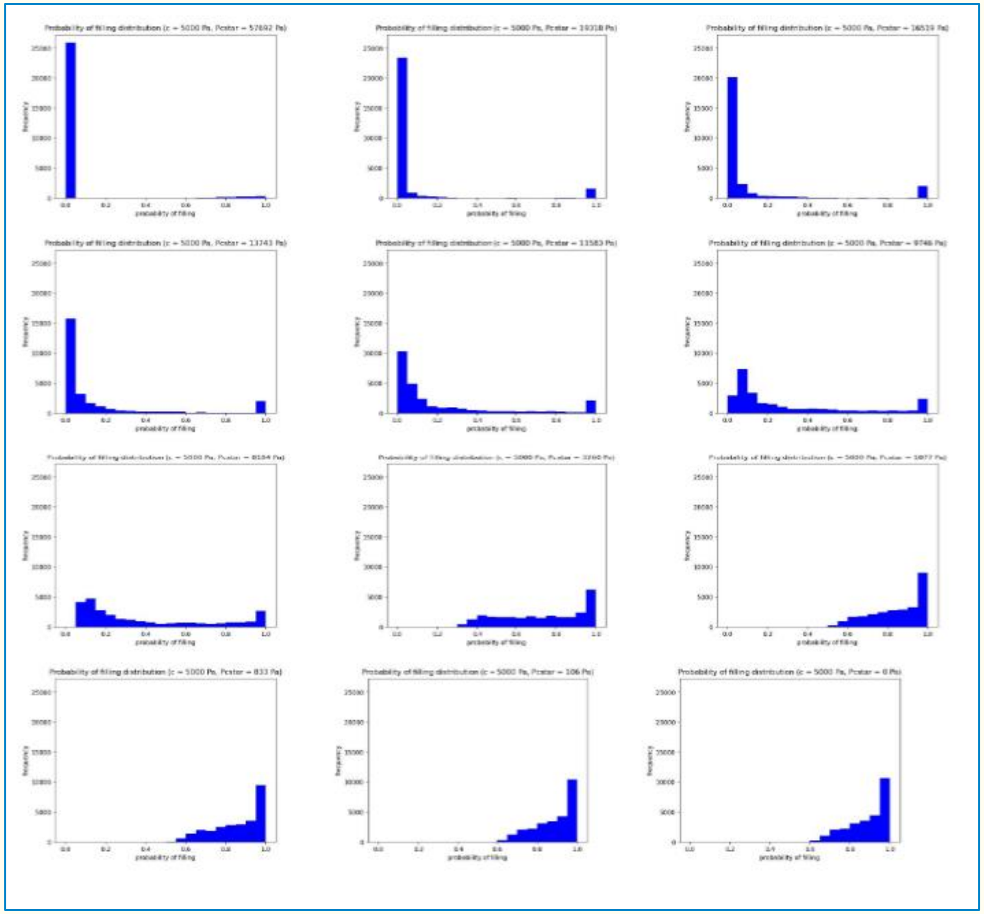
Simulation results

Probability filling when c = 5000 Pa



Plots of Pc, kr and fw against Sw



Probability distribution for filling at the different values of P_c*

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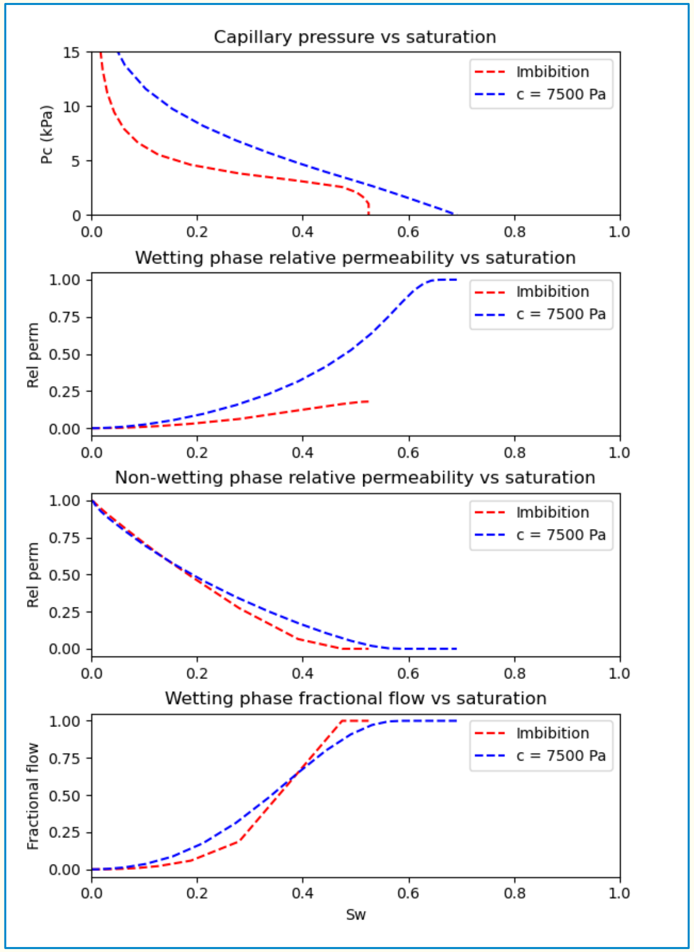
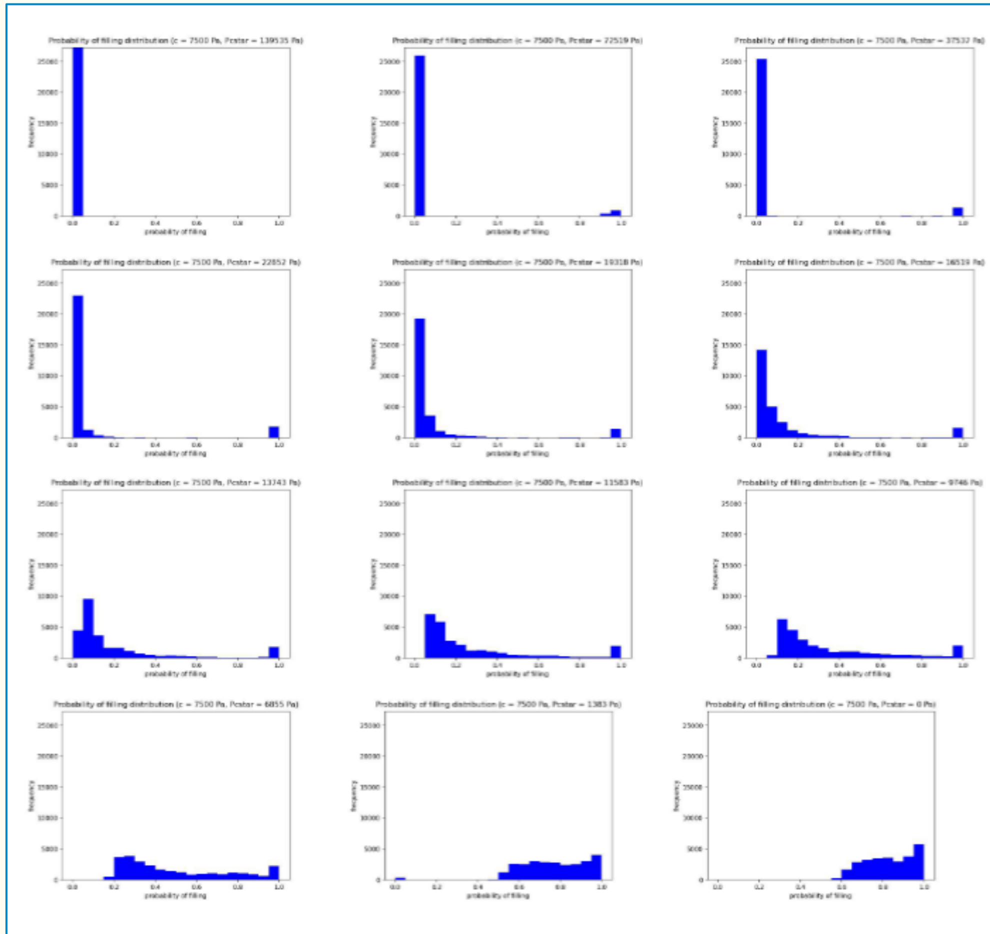
Simulation results

Probability filling when c = 7500 Pa



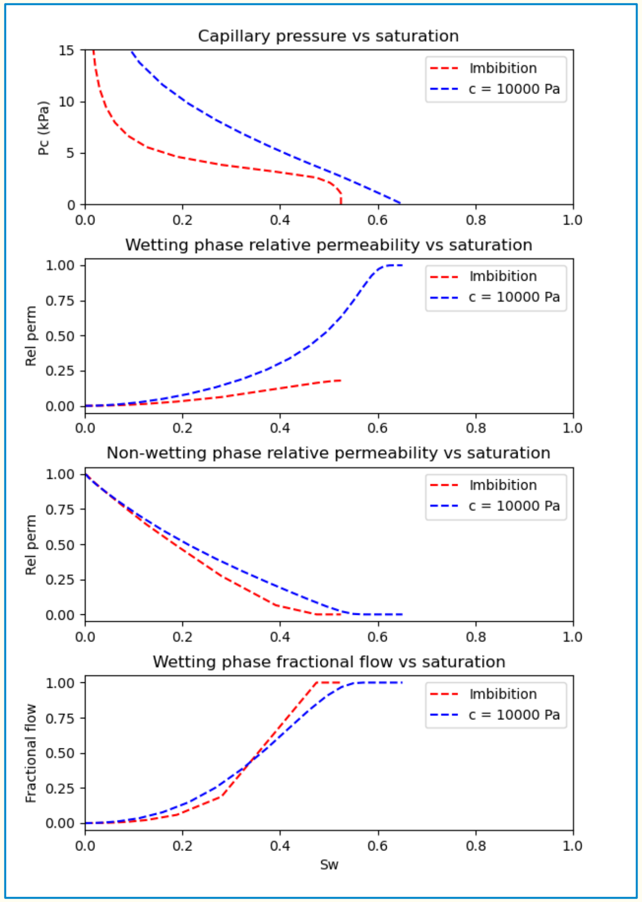
Plots of Pc, kr and fw against Sw



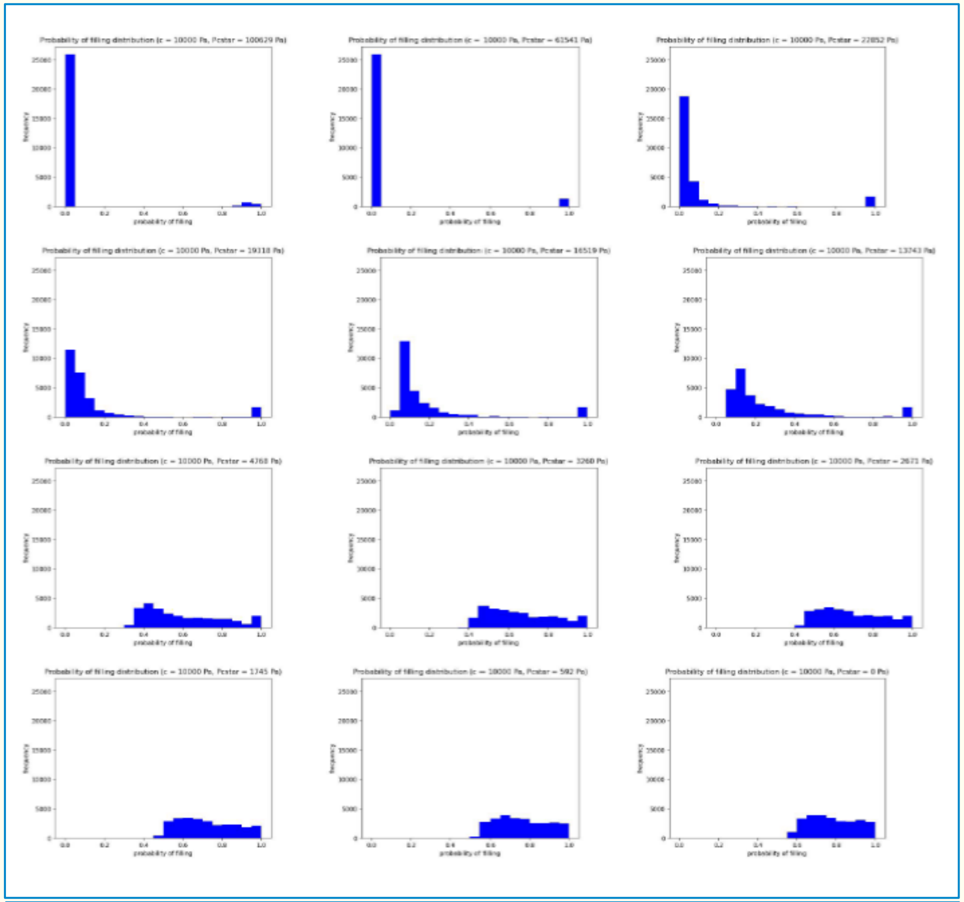
Probability distribution for filling at the different values of P_c*

Simulation results

Probability filling when c = 10000 Pa



Plots of Pc, kr and fw against Sw



Probability distribution for filling at the different values of P_c*

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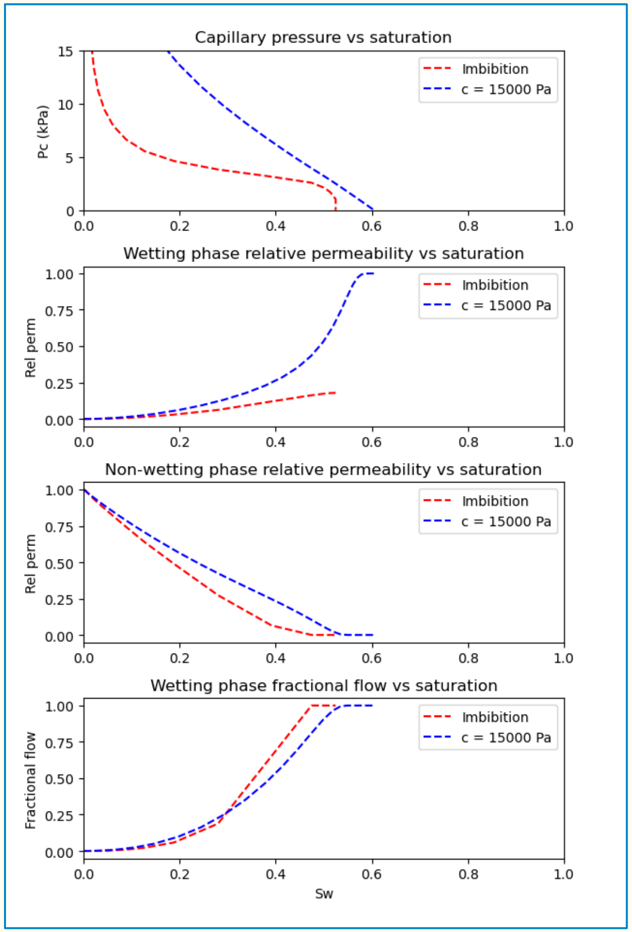
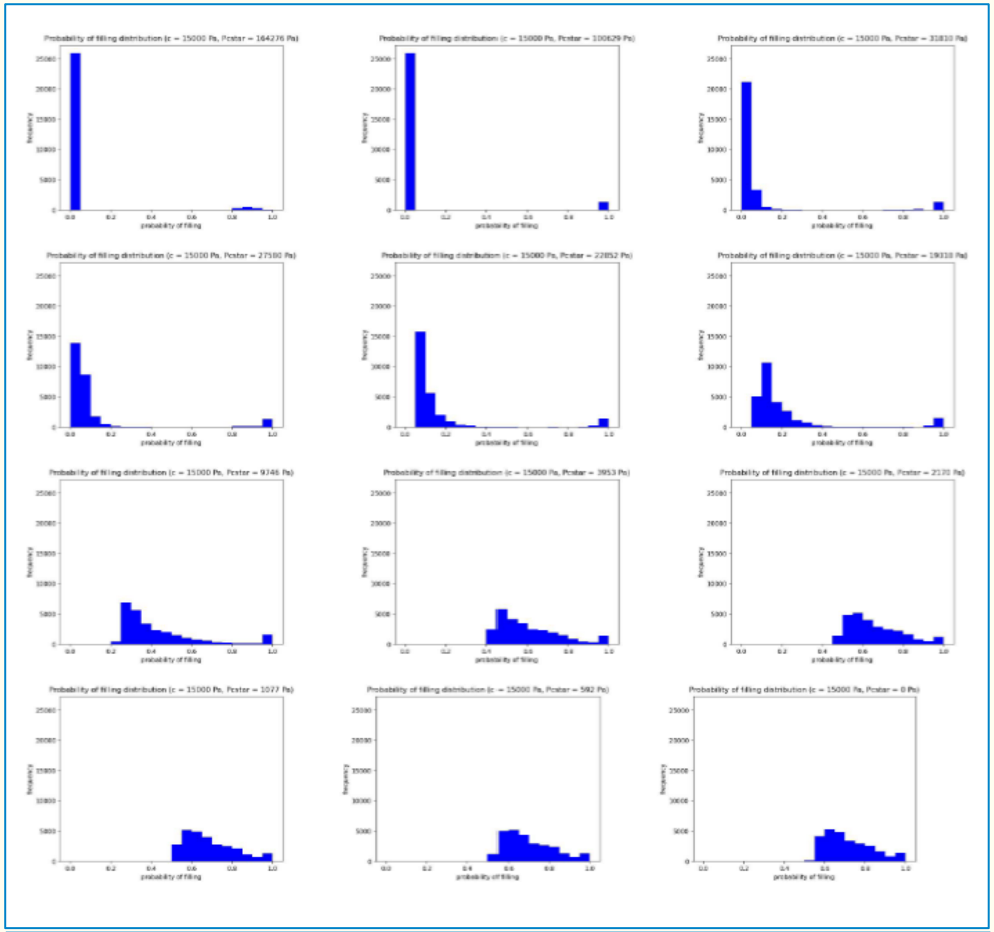
Simulation results

Probability filling when c = 15000 Pa



Plots of Pc, kr and fw against Sw



Probability distribution for filling at the different values of P_c*

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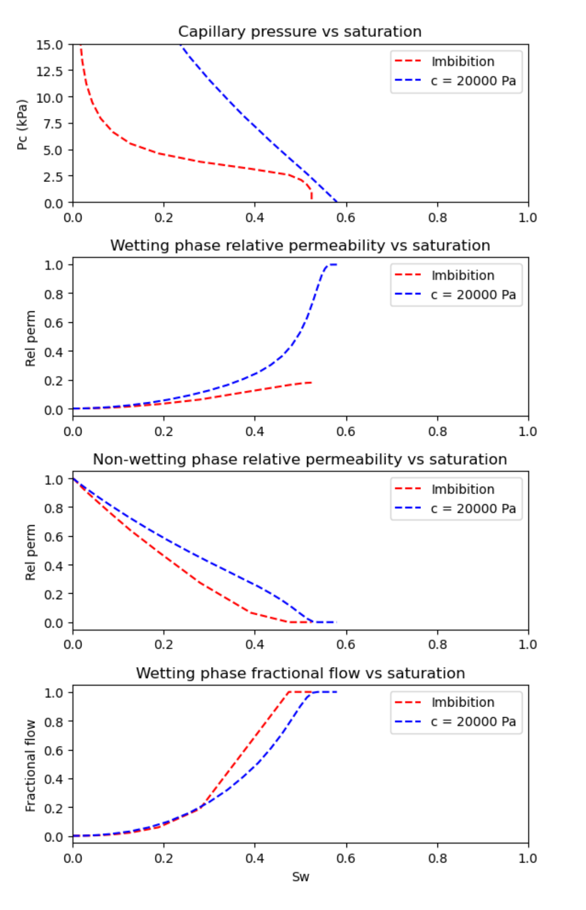
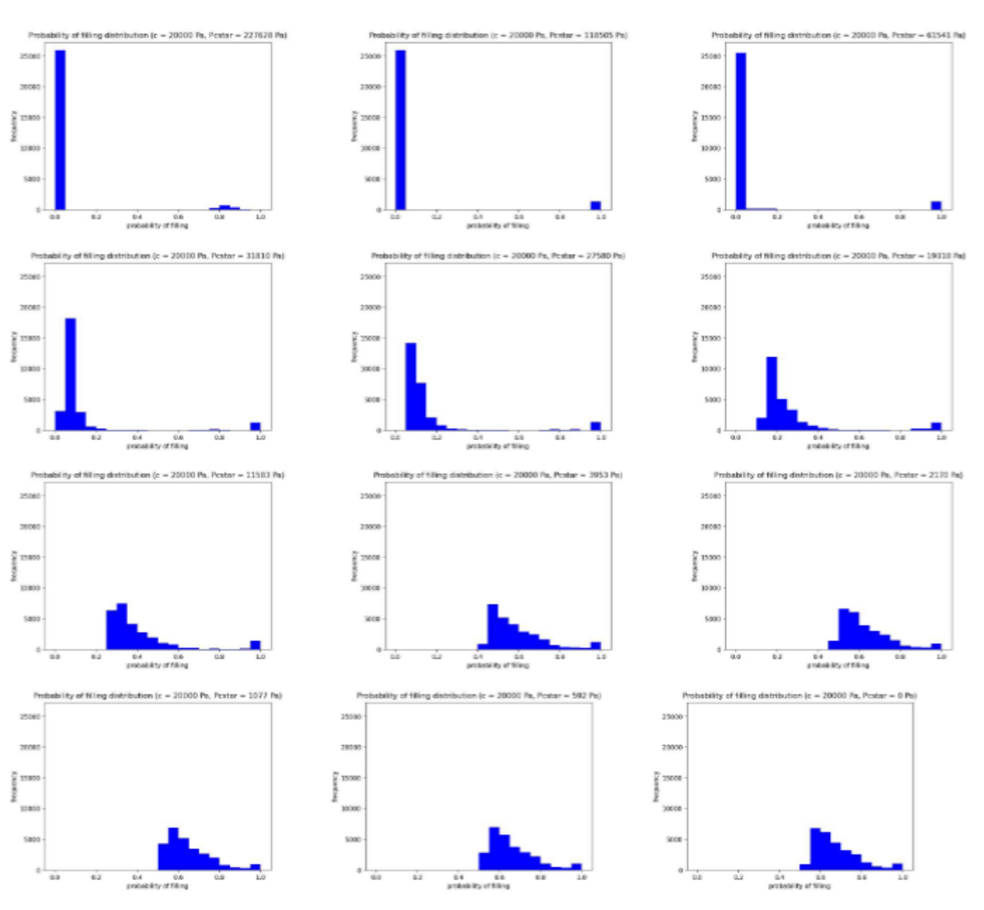
Simulation results

Probability filling when c = 20000 Pa



Plots of Pc, kr and fw against Sw



Probability distribution for filling at the different values of P_c*

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References

Gesho, M. (2017). Dynamic Pore Network Modeling of Two-Phase Flow Through Fractured Porous Media : Direct Pore-to-Core Upscaling of Displacement Processes.

Hansen, A., Flekkoy, E.G., Sinha, S. and Slotte, P.A. (2022). A Statistical Mechanics for Immiscible and Incompressible two-phase Flow in Porous Media.

Jayakumar, A., Singamneni, S., Ramos, M.V., Al-Jumaily, A.M., & Pethaiah, S.S. (2017). Manufacturing the Gas Diffusion Layer for PEM Fuel Cell Using a Novel 3D Printing Technique and Critical Assessment of the Challenges Encountered. *Materials, 10*.

Raeini, A.Q., Blunt, M.J. & Bijeljic, B. (2012). Modelling two-phase flow in porous media at the pore scale using the volume-of –fluid method. *Journal of Computational Physics*, 231(17).

Spurin, C., Bultreys, T., Bijeljic, B., Blunt, M.J. & Krevor, S. (2019). Intermittent Fluid Connectivity during two-phase Flow in a heterogenous Carbonate Rock. Physical Review E, 100.

Tallakstad, K.T., Knudsen, H.A., Ramstad, T., Lovoll, G., Maloy, K.J., Toussaint, R. and Flekkoy, E.G (2009). Steady-state two-phase flow in Porous Media: Statistics and Transport Properties. Physical Review Letters, 102.

Tallakstad, K.T., Lovoll, G., Knudsen, H.A., Ramstad, T., Flekkoy, E.G. , Maloy, K.J., and Toussaint, R. (2009). Steady-state two-phase flow in Porous Media: An Experimental Study. Physical Review Letters, 80.

Zhang, Y., Bijeljic, B., Gao, Y., Lin, Q. and Blunt, M.J. (2021). Quantification of Nonlinear Flow in Porous Media. *Geophysical Research Letters*, 48.