

Ignacio Manzur

Undergraduate student, Year 2 in 2019-20, BSc Mathematics (Pure Mathematics), Dept of Mathematics

UROP: Summer 2020 (undertaken in the Dept of Mathematics)

A perspective: Arithmetic Geometry and the Weil Conjectures

“Tell me and I forget, teach me and I remember, involve me and I learn”. It would be quite foolish to expect to make progress in Pure Mathematics without getting into it head on. However, when studying more advanced material, often times you are alone. That is why being able to have feedback and guidance from a professor at Imperial during a UROP was a real chance. My supervisor was my Analysis II professor in Year 2, but I knew that his research interests were very close to the area of mathematics I had wanted to study for a long time: Algebraic Geometry.

We quickly agreed to start first with the Weil conjectures for projective curves as an introduction to this vast field. This took me my whole UROP to complete in the end. But it has been a crucial moment of intellectual development. The theory is so extensive that it can feel quite daunting. Lorenzini's *Invitation to Arithmetic Geometry*, that we closely followed, is a very big book. Atiyah's *Introduction to Commutative Algebra*, which gave me the necessary Algebra background, can be hard at times. But somehow after getting lost for a while in this deep and dark forest, you find your way out. This project reinforced my resilience and my strength of will. More importantly it has made me change my conception of mathematics. One would think that the more theory one knows, the more it becomes easy to solve complex problems. It is actually not exactly true. Whenever there are no more clear guidelines as in a university module for example, you are pretty much alone against the problem. The way to link it to the theory you know is an equally (if not more) important part of the resolution but does not depend directly on the amount of theory you have accumulated. A balance between intuition and knowledge, that is what is needed.

Obviously, the situation has not been helpful, but it has not been fundamentally important. My supervisor and I have been in different countries for most of the project. That means that I could not get immediate answers to my questions, which has been as beneficial as it has been detrimental. Evidently, I have been confronted to my questions for a longer time, which often times has had me think twice before asking them, and eventually answering them myself. But other times when I had a doubt about something like notation, or a mistake in a book, it has slowed my progression. Speaking to it to my personal tutor at Imperial, he told me jokingly that I had experienced a 'mini PhD', so I guess it has all been a preparation for what is to come after all. I consider myself to be incredibly lucky, despite the situation, to have been able to build an excellent relationship with my supervisor. Effectively the UROP has extended into a 6-months project, which will continue until the end of my third year. We will look into Schemes and Etale Cohomology, which I have just checked by reading my UROP expectations, was what I wanted to get to.

My supervisor is now one of my referees for my applications to Master's programs. This is a crucial part of what the UROP has brought me, because I know that my supervisor will be able to precisely depict and communicate my passion for mathematics, the theory that we have covered, and the way that I effectively 'do' mathematics. There are not many opportunities for undergraduates to be able to show to a professor what they are able to produce and comprehend, but a UROP is one of them.